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Models and Relations in Economics and Econometrics

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Abstract

Based on a money market analysis the paper discusses possible pitfalls in macroeconomic inference related to inadequate stochastic model formulation. A number of questions related to concepts such as empirical and theoretical steady-states, speed of adjustment, feed-back and interaction effects, and driving forces are addressed within the framework of the cointegrated VAR model. The economic notion of anticipated and unanticipated shocks to a system is discussed from an econometric point of view.

Keywords: I(2), price homogeneity, money market, Cointegrated VAR

1. Introduction[1]

Empirical macroeconomic models based on a probability approach (Haavelmo, 1943) explicitly starts from a stochastic formulation of the chosen data. Because most macroeconomic data exhibit strong time dependence, it is natural to formulate the empirical model in terms of time dependent stochastic processes. Within this family the vector autoregressive (VAR) process based on Gaussian errors has shown to be a popular choice. There are many reasons for this: the VAR model is flexible, easy to estimate, and it usually gives a good fit to macroeconomic data. However, the possibility of combining long-run and short-run information in the data by exploiting the cointegration property is probably the most important reason why the cointegrated VAR model continues to receive the interest of both econometricians and applied economists.

Theoretical economic models, on the other hand, have traditionally been developed as non-stochastic mathematical entities and applied to empirical data by adding a stochastic error process to the mathematical model. As an example of this approach I will use the macroeconomic treatment in "Inflation and Monetary Policy", Chapter 9 in D. Romer (1996): "Advanced Macroeconomics".

From an econometric point of view the two approaches are fundamentally different: one starting from an explicit stochastic formulation of <u>all</u> data and then <u>reducing</u> the general statistical (dynamic) model by imposing testable restrictions on the parameters, the other starting from a mathematical (static) formulation of a theoretical model and then <u>expanding</u> the model by adding stochastic components. For a detailed methodological discussion of the two approaches, see for example Gilbert (1986), Hendry (1995), Juselius (1993), and Pagan (1987).

Unfortunately, the two approaches have shown to produce very different results even when applied to identical data and, hence, different conclusions. From a scientific point of view this is not satisfactory. Therefore, I will attempt to bridge the gap between the two views by starting from some typical questions of theoretical interest and then show how one would answer these questions based on a *statistical* analysis of the VAR model. Because the latter by construction is "bigger" than the theory model, the empirical analysis not only answers a specific theoretical question, but also gives additional insight into the macroeconomic

[1] This paper has benefitted from useful comments by Carlo Favero, Kevin Hoover, Søren Johansen, Adrian Pagan, and the participants at the conference "Theory and Evidence in Macroeconomics" in Bergamo. Financial support from the Joint Committee of the Nordic Social Sciences Research Council is gratefully acknowledged.

problem.

A theory model can be simplified by the *ceteris paribus* assumptions "everything else unchanged", whereas a statistically well-specified empirical model has to address the theoretical problem in the context of "everything else changing". By embedding the theory model in a broader empirical framework, the analysis of the statistically based model can provide evidence of possible pitfalls in macroeconomic reasoning. In this sense the VAR analysis can be useful for generating new hypotheses, or for suggesting modifications of too narrowly specified theoretical models. As a convincing illustration see Hoffman (1999).

All through the paper I will address questions of empirical relevance for the analysis of monetary inflation and for the transmission mechanisms of monetary policy. These questions have been motivated by many empirical VAR analyses of money, prices, income, and interest rates and include questions such as:

• How effective is monetary policy when based on changes in money stock or changes in interest rates?

• What is the effect of expanding money supply on prices in the short run? in the medium run? in the long run?

• Is an empirically stable demand for money relation a prerequisite for monetary policy control to be effective?

• How strong is the direct (indirect) relationship between a monetary policy instrument and price inflation?

Based on the VAR formulation I will demonstrate that every empirical statement can, and should, be checked for its consistency with all previous empirical and theoretical statements. This is in contrast to many empirical investigations, where inference relies on many untested assumptions using test procedures that only make sense in isolation, but not in the full context of the empirical model.

The organization of the paper is as follows: In Section 2 I briefly consider the of treatment of inflation and monetary policy in Romer (1996) with special reference to the equilibrium in the money market. I discuss the problem of inverting an equilibrium money demand relation to get a price relation when the model is stochastic. Section 3 discusses informally some empirical and theoretical implications of unit roots in the data and Section 4 addresses more formally a stochastic formulation based on a decomposition of the data into trends, cycles, and irregular components. Section 4.1 gives an empirical motivation for treating the stochastic trend in nominal prices as $I(2)$, and Section 4.2 as $I(1)$. The consequences of either choice is spelt out and the implications for testing long-run and medium-run price homogeneity are discussed. Section 5 introduces the VAR

model and defines the $I(2)$ and $I(1)$ models as parameter restrictions on the general model. Section 6 discusses the choice of cointegration rank given *a priori* economic knowledge. Section 7 discusses just- and over-identifying restrictions on the long-run parameters and gives examples of restrictions which are consistent with (i) real and nominal separation and (ii) real and nominal interactions in the economy. Section 8 discusses identification of common driving trends from a statistical and economic point of view. The possibility of identifying and estimating economic shocks based on estimated residuals is discussed at some length. Section 9 treats identification of the short-run adjustment parameters given an identified long-run structure and discusses some questions relevant for monetary policy. Section 11 summarizes and concludes.

2. Inflation and money growth

A fundamental proposition in macroeconomic theory is that growth in money supply in excess of real productive growth is the cause of inflation, at least in the long run. I will consider briefly some conventional ideas underlying this belief as described in Chapter 9 by Romer (1996).

The well-known diagram illustrating the intersection of aggregate demand and aggregate supply provides the framework for identifying potential sources of inflation as shocks shifting either aggregate demand upwards or aggregate supply to the left. See the upper panel of Figure 2.1.

As examples of aggregate supply shocks that shift the AS curve to the left Romer (1996) mentions; negative technology shocks, downward shifts in labor supply, upwardly skewed relative-cost shocks. As examples of aggregate demand shocks that shift the AD curve to the right he mentions; increases in money stock, downward shifts in money demand, increases in government purchases. Since all these types of shocks, and many others, occur quite frequently there are many factors that potentially can affect inflation. Some of these shocks may only influence inflation temporarily and are, therefore, less important than shocks with a permanent effect on inflation. Among the latter economists usually emphasize changes in money supply as the crucial inflationary source. The economic intuition behind this is that other factors are limited in scope, whereas money in principle is unlimited in supply.

More formally the reasoning is based on money demand and supply and the condition for equilibrium in the money market:

$$M/P = L(R, Y), \quad L_R < 0, \;\; L_y > 0. \tag{2.1}$$

where M is the money stock, P is the price level, R the nominal interest rate, Y real income, and $L(\cdot)$ the demand for real money balances. Based on the equilibrium condition, i.e. no changes in any of the variables, Romer (1996) concludes that the price level is determined by:

$$P = M/L(R, Y) \tag{2.2}$$

The equilibrium condition (2.1) and, hence (2.2), is a static concept that can be thought of as a hypothetical relation between money and prices for fixed income and interest rate. The underlying comparative static analysis investigates the effect on one variable, say price, when changing another variable, say money supply, with the purpose of deriving the new equilibrium position after the change. Thus, the focus is on the hypothetical effect of a change in one variable (M) on another variable (P), when the additional variables (R and Y) are exogenously given and everything else is taken account of by the *ceteris paribus* assumption.

However, when time is introduced the *ceteris paribus* assumption and the assumption of fixed exogenous variables become much more questionable. Neither interest rates nor real income have been fixed or controlled in most periods subject to empirical analysis. Therefore, in empirical macroeconomic analysis <u>all variables</u> (inclusive the *ceteris paribus* ones) are more or less continuously subject to shocks, some of which permanently change the previous equilibrium condition. In this sense an equilibrium position is an inherently time dependent concept in empirical modelling. Hence, the static equilibrium concept has to be replaced by a dynamic concept, for instance a steady-state position. For an equilibrium relation time is irrelevant. Discussing a steady-state relation without a time index is meaningless.

In a typical macroeconomic system new disturbances push the variables away from steady-state, but the economic adjustment forces pull them back towards a new steady-state position. The adjustment back to steady-state is disturbed by new shocks and the system essentially never comes to rest. Therefore, we will not be able to observe a steady-state position and the empirical investigation has to account for the stochastic properties of the variables as well as the theoretical equilibrium relationship between them. See the lower panel of Figure 2.1 for an illustration of a stochastic steady-state relation.

In (2.1) the money market equilibrium is an exact mathematical expression and it is straightforward to invert it to determine prices as is done in (2.2). The

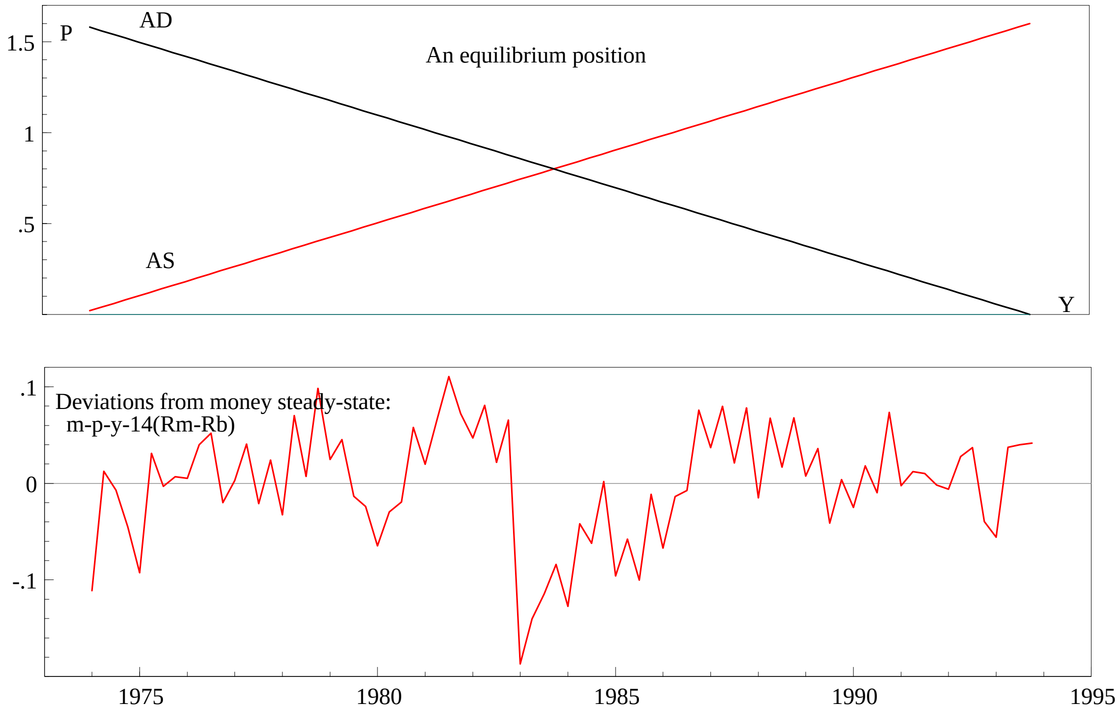

Figure 2.1: An equilibrium position of the AD and AS curve (upper panel) and deviations from an estimated money demand relation for Denmark: $(m-p-y)_t - 14.1(R_m - R_b)$ (lower panel).

observations from a typical macroeconomic system is adequately described by a stochastic vector time series process. But in stochastic systems, inversion of (2.1) is no longer guaranteed (see for instance Hendry and Ericsson, 1991) and is likely to result in misleading conclusions.

Because the observed money stock is not a measurement of an equilibrium position it can be demand or supply determined or both. This raises the question whether it is possible to empirically identify and estimate the underlying theoretical relations. For instance, if central banks are able to effectively control money stock, then observed money holdings are likely to be supply determined and the demand for money has to adjust to the supplied quantities. This is more likely to be the case in trade and capital regulated economies or in economies with with flexible exchange rates, whereas in open deregulated economies with fixed exchange rates central banks would not in general be able to control money stock. In the latter case one would expect observed money stock to be demand determined.

Given that the money demand relation can be *empirically* identified, the statistical estimation problem has to be addressed. Because macroeconomic variables are generally found to be nonstationary, standard regression methods are no longer feasible from an econometric point of view. But since cointegration analysis specifically addresses the nonstationarity problem, it is a feasible solution in this respect. The empirical counterpart of (2.1) can be written as a cointegrating relation, i.e.:

$$(M/P)_t - L(R_t, Y_t) = v_t \tag{2.3}$$

where v_t is a stationary process measuring the deviation from the steady-state position at time t. The stationarity of v_t implies that whenever the system has been shocked it will adjust back to equilibrium. This is illustrated in Figure 2.1 (lower panel) where the deviations from an estimated money demand relation based on Danish data (Juselius, 1998b) is graphed. Note the large equilibrium error at about 1983, as a result of removing restrictions on capital movements and the consequent adjustment back to steady-state.

However, empirical investigation of (2.3) based on cointegration analysis poses several additional problems. Although in a theoretical exercise it is straightforward to keep some of the variables fixed (the exogenous variables), in an empirical model none of the variables in (2.1), i.e. money, prices, income or interest rates, can be assumed to be fixed (i.e. controlled). The stochastic feature of all variables implies that the equilibrium adjustment can take place in either money, prices, income or interest rates. Therefore, the equilibrium deviation v_t is not necessarily due to a money supply shock at time t, but can originate from any change in the variables. Hence, it is no longer possible to interpret a coefficient in a cointegrating relation as in the conventional regression context, which is based on the assumption of "fixed" regressors. In multivariate cointegration analysis all variables are stochastic and a shock to one variable is transmitted to all other variables via the dynamics of the system until the system has found its new equilibrium position.

The empirical investigation of the above questions raises several econometric questions:

What is the meaning of a shock and how do we measure it econometrically? How do we distinguish empirically between the long run, the medium run and the short run? Given the measurements can the parameter estimates be given an economically meaningful interpretation? These questions will be discussed in more detail in the subsequent sections.

3. The time dependence of macro data

As advocated above, the strong time dependence of macroeconomic data suggests a statistical formulation based on stochastic processes. In this context it is useful to distinguish between:

- stationary variables with a short time dependence and
- nonstationary variables with a long time dependence.

In practise, we classify variables exhibiting a high degree of time persistence (insignificant mean reversion) as nonstationary and variables exhibiting a significant tendency to mean reversion as stationary. However I will argue that the stationarity/nonstationarity or, alternatively, the order of integration of a variable, is not in general a *property of an economic variable* but a convenient statistical approximation to distinguish between the short-run, medium-run, and long-run variation in the data. I will illustrate this with a few examples involving money, prices, income, and interest rates.

Most countries have exhibited periods of high and low inflation, lasting sometimes a decade or even more, after which the inflation rate has returned to its mean level. If inflation crosses its mean level, say ten times, the econometric analysis will find significant mean reversion and hence conclude that inflation rate is stationary. For this to happen we might need up to hundred years of observations. The time path of, for example, quarterly European inflation over the last few decades will cover a high inflation period in the seventies and beginning of the eighties and a low inflation period from mid-eighties until the present date. Crossing the mean level a few times is not enough to obtain statistically significant mean reversion and the econometric analysis will show that inflation should be treated as nonstationary. This is illustrated in Figure 3.1. where yearly observations of the Danish inflation rate has been graphed for 1901-1992 (upper panel), for 1945-1992 (middle panel), and 1975-1992 (lower panel). The first two time series of inflation rates look mean-reverting (though not to zero mean inflation), whereas significant mean-reversion would not be found for the last section of the series.

That inflation is considered stationary in one study and nonstationary in another, where the latter is based, say, on a sub-sample of the former might seem

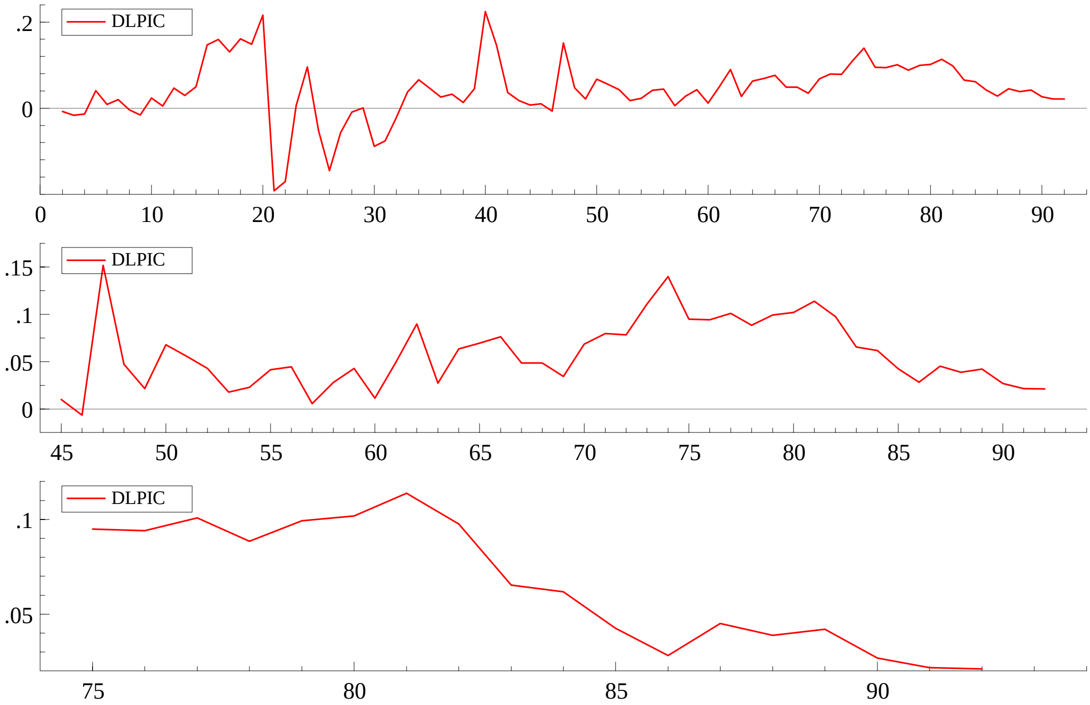

Figure 3.1: Yearly Danish inflation 1901-92 (upper panel), 1945-92 (middle panel), and 75-92 (lower panel).

contradictory. This need not be so, unless a unit root process is given a structural economic interpretation. There are many arguments in favor of considering a unit root (a stochastic trend) as a convenient econometric approximation rather than as a deep structural parameter. For instance, if the time perspective of our study is the macroeconomic behavior in the medium run, then most macroeconomic variables exhibit considerable inertia, consistent with nonstationary rather than stationary behavior. Because inflation, for example, would not be statistically different from a nonstationary variable, treating it as a stationary variable would invalidate the statistical analysis and, therefore, lead to wrong economic conclusions. On the other hand, treating inflation as a nonstationary variable gives us the opportunity to find out which other variable(s) have exhibited a similar stochastic trend by exploiting the cointegration property. This will be discussed

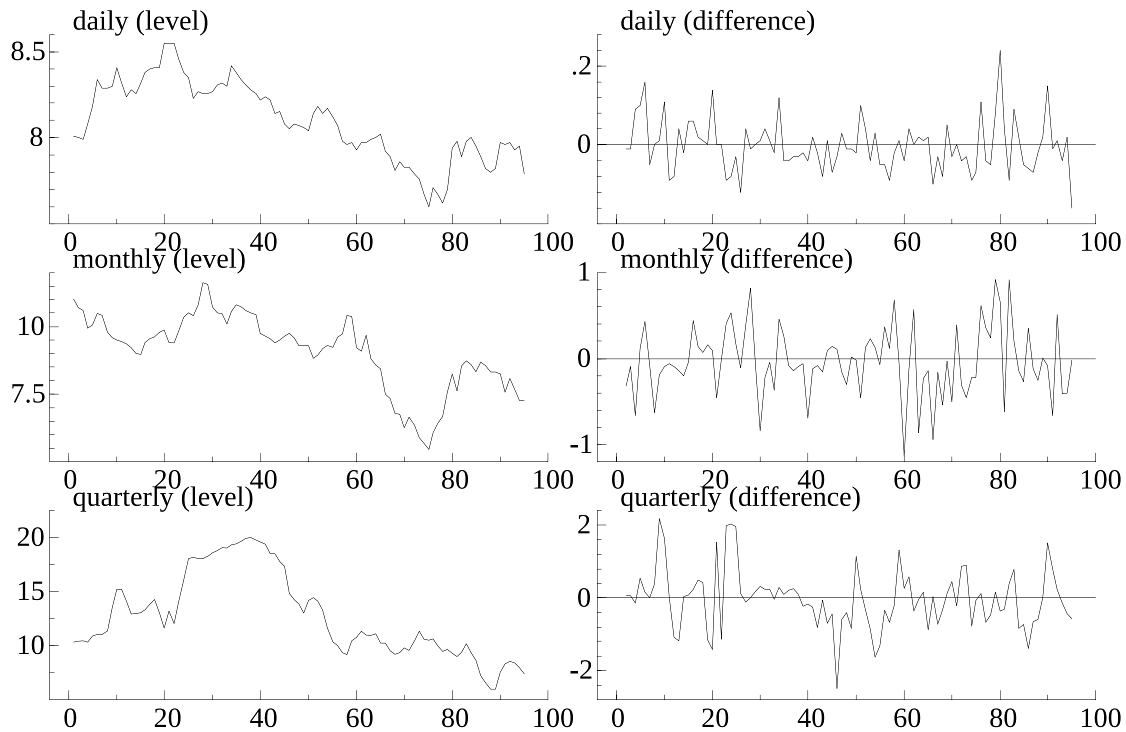

Figure 3.2: Average Danish bond rates, based on daily observations, 1.5.-25.9.95 (upper panel), monthly observations, Nov 1987 - Sept 1995 (middle panel), and quarterly observations, 1972:1-1995:3, (lower panel).

at some length in Section 4, where I will demonstrate that the unit root property of economic variables is very useful for the empirical analysis of long- and medium-run macroeconomic behavior.

When the time perspective of our study is the long historical macroeconomic movements, inflation as well as interest rates are likely to show significant mean reversion and, hence, can be treated as a stationary variable.

Finally, to illustrate that the same type of stochastic processes are able to adequately describe the data, independently of whether one takes a close-up or a long-distance look, I have graphed the Danish bond rate in levels and differences based on a sample of 95 quarterly observations (1972:1-1995:3), 95 monthly observations (1987:11-1995:9), and 95 daily observations (1.5.95-25.9.95). The daily sample corresponds to the little hump at the end of the quarterly time series. It would be considered a small stationary blip from a quarterly perspective, whereas from a daily perspective it is nonstationary, showing no significant mean reversion. Altogether, the three time series look very similar from a stochastic point of view.

Thus, econometrically it is convenient to let the definition of long-run or short-run, or alternatively the very long-run, the medium long-run, and the short-run, depend on the time perspective of the study. From an economic point of view the question remains in what sense can a "unit root" process be given a "structural" interpretation.

4. A stochastic formulation

To be able to discuss the above questions I will consider a conventional decomposition into trend, T, cycle, C, and irregular component, I, of a typical macroeconomic variable.

$$X = T * C * I$$

Instead of treating the trend component as deterministic, as is usually done in conventional analysis, I will allow the trend to be both deterministic, T_d, and stochastic, T_s, i.e. $T = T_s \times T_d$, and the cyclical component to be of long duration, say 6-10 years, C_l, and of shorter duration, say 3-5 years, C_s, i.e. $C = C_l \times C_s$. The reason for distinguishing between short and long cycles is that a long/short cycle can either be treated as nonstationary or stationary depending on the time perspective of the study. As an illustration of long cycles that have been found nonstationary by the statistical analysis (Juselius, 1998b) see the graph of trend-adjusted real income in Figure 4.1, middle panel.

An additive formulation is obtained by taking logarithms:

$$x = (t_s + t_d) + (c_l + c_s) + i \tag{4.1}$$

where lower case letters indicate a logarithmic transformation. Though the stochastic time dependency of the variables are of primary interest in the subsequent discussions, the linear time trend cannot be left out since it is a measure of average linear growth trends usually present in economic data.

To give the economic intuition for the subsequent multivariate cointegration analysis of money demand / money supply relations, I will illustrate the ideas in

Section 4.1 and 4.2 using the time series vector $\mathbf{x}_t = [m, p, y, R_m, R_b]_t$, $t = 1, ..., T$, where m is a measure of money stock, p the price level, y real income, R_m the own interest on money stock, and R_b the interest rate on bonds. All variables are treated as stochastic and, hence, from a statistical point of view need to be modelled, independently of whether they are considered endogenous or exogenous in the economic model.

To illustrate the ideas I will assume two autonomous shocks, u_1 and u_2, where for simplicity u_1 is assumed to be a shock causing a permanent shift in the AD curve and u_2 a shock causing a permanent shift in the AS curve. This case would be consistent with a vertical aggregate supply (AS) curve and downward sloping aggregate demand (AD) curve.

To clarify the connection between the econometric analysis and the economic interpretation I will first assume that the empirical analysis is based on a quarterly model of, say, a few decades and then on a yearly model of, say, a hundred years. In the first case, when the perspective of the study is the medium run, I will argue that prices should generally be treated as $I(2)$, whereas in the latter case, when the perspective is the very long run, prices can sometimes be approximated as a strongly correlated $I(1)$ process.

Figure 4.1 illustrates different stochastic trends in the Danish quarterly data. The stochastic $I(2)$ trend in the upper panel corresponds to trend-adjusted prices, the stochastic $I(1)$ trend in the middle panel corresponds to trend-adjusted real income, and the stochastic $I(1)$ trend in the lower panel corresponds to the inflation rate, i.e. to the differenced the $I(2)$ trend.

The concept of a common stochastic trend or a driving force requires a further distinction between:

- an unanticipated shock with a permanent effect (a disturbance to the system with a long lasting effect)
- an unanticipated shock with a transitory effect (a disturbance to the system with a short duration).

To give the non-expert reader a more intuitive understanding for the meaning of a stochastic trend of first or second order, I will describe inflation π_t as the sum of permanent, ε_{p_t}, and transitory, ε_{s_t}, shocks, starting from an initial time point π_0, i.e.:

$$\pi_t = \varepsilon_{pt} + \varepsilon_{pt-1} + \varepsilon_{pt-2} + ... + \varepsilon_{p1} + \varepsilon_{st} + \varepsilon_{st-1} + \varepsilon_{st-2} + ... + \varepsilon_{s1} + \pi_0$$

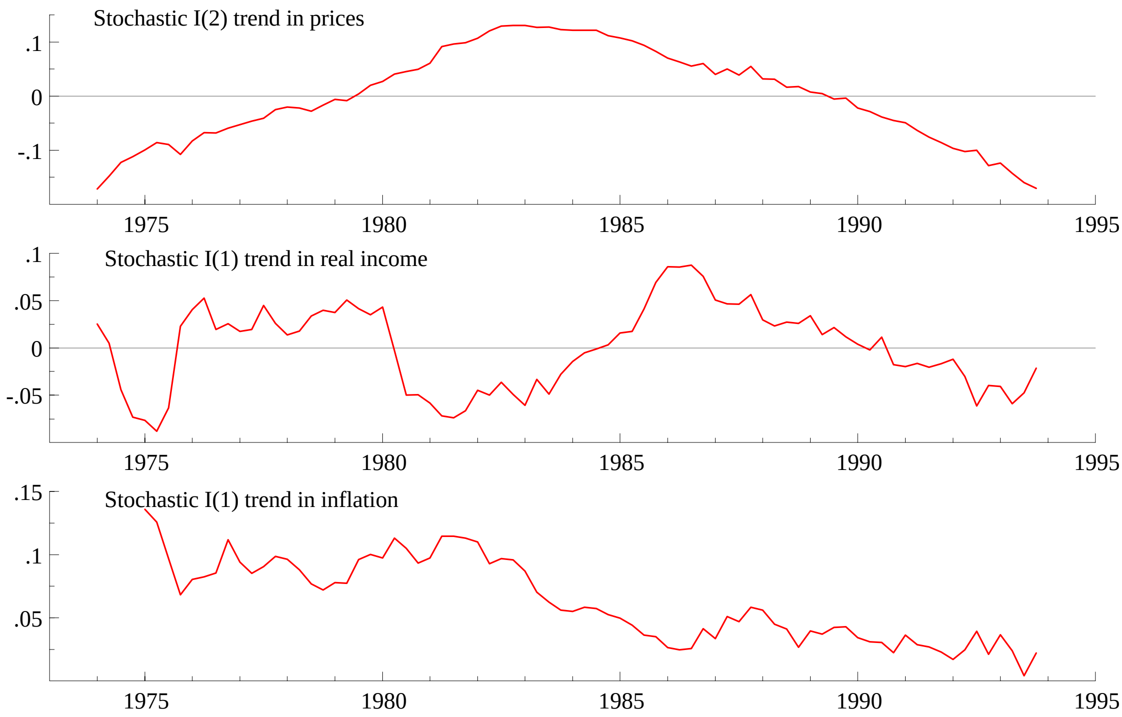

Figure 4.1: Stochastic trends in Danish prices, real income and inflation, based on quarterly data 1975:1-1994:4

A permanent shock is by definition a shock that has a lasting effect on the level of inflation, such as a permanent increase in government expenditure, whereas the effect of a transitory shock disappears either during the next period or gradually. An example of a transitory price shock is a value added tax imposed in one period and removed the next. In the latter case prices increase temporarily, but return to their previous level after the removal. Therefore, a transitory shock can be described as a shock that occurs a second time in the series but then with the opposite sign. Hence, a transitory shock disappears in cumulation, whereas a permanent shock has a long lasting effect on the level. In practise we only observe one shock being the sum of the two, i.e. $\varepsilon_t = \varepsilon_{pt} + \varepsilon_{st}$. However, in the summation:

$$\pi_t = \Sigma_{i=1}^{t} \varepsilon_i + \pi_0 \tag{4.2}$$

only the permanent shocks will have a lasting effect and we call $\Sigma_{i=1}^{t}\varepsilon_i$ a stochastic trend. The difference between a linear stochastic and deterministic trend is that increments of a stochastic trend change randomly, whereas those of a deterministic trend are constant over time. This is illustrated in the lower panel of Figure 4.1.

A representation of prices instead of inflation is obtained by integrating (4.2) once, i.e.

$$p_t = \Sigma\pi_i = \Sigma_{s=1}^{t}\Sigma_{i=1}^{s}\varepsilon_i + \pi_0 t + p_0. \tag{4.3}$$

It appears that inflation being $I(1)$ with a nonzero mean, corresponds to prices being $I(2)$ with linear trends. The stochastic $I(2)$ trend is illustrated in the upper part of Figure 4.1.

The question whether inflation rates should be treated as $I(1)$ or $I(0)$ has been subject to much debate. Figure 3.1 illustrated that inflation measured over the last decades was probably best approximated by a nonstationary process, whereas measured over a century by a stationary, though strongly autocorrelated, process. For a description of the latter case (4.2) should be replaced with:

$$\pi_t = \Sigma_{i=1}^{t}\rho^i\varepsilon_i + \pi_0 \tag{4.4}$$

where the autoregressive parameter ρ is less than but close to one. In this case prices would be represented by:

$$p_t = \Sigma\pi_i = \Sigma_{s=1}^{t}\Sigma_{i=1}^{s}\rho^i\varepsilon_i + \pi_0 t + p_0 \tag{4.5}$$

i.e. by a strongly autoregressive first order stochastic trend and a deterministic linear trend.

The difference between (4.2) and (4.4) is only a matter of approximation. In the first case the parameter ρ is approximated with unity, because the sample period is too short for the estimate to be statistically different from one. In the second case the sample period contains enough turning points for ρ to be significantly different from one. I will argue below that, unless a unit root is given a structural interpretation, the choice of one representation or the other is not very important as such, as long as the economic and econometric analyses are consistent with each other.

4.1. Treating prices as I(2)

In this section I will assume that the long-run stochastic trend t_s in (4.1) can be described by the twice cumulated AD shocks, $\Sigma\Sigma u_{1i}$, and the long cyclical component c_l by the once cumulated AD shocks, Σu_{1i}, and the once cumulated AS shocks, Σu_{2i}. This representation gives us the possibility of distinguishing between the long-run stochastic trend component in prices, $\Sigma\Sigma u_{1i}$, the medium-run stochastic trend in price inflation, Σu_{1i}, and the medium-run stochastic trend in real activity, Σu_{2i}.

As an illustration of how the econometric analysis is influenced by the above assumptions I will consider the following decomposition of the data vector:

$$\begin{bmatrix} m_t \\ p_t \\ y_t \\ R_{mt} \\ R_{bt} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{21} \\ 0 \\ 0 \\ 0 \end{bmatrix} [\Sigma\Sigma u_{1i}] + \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \\ d_{41} & d_{42} \\ d_{51} & d_{52} \end{bmatrix} \begin{bmatrix} \Sigma u_{1i} \\ \Sigma u_{2i} \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ 0 \\ 0 \end{bmatrix} [t] + \text{stat.comp.} \tag{4.6}$$

The deterministic trend component, $t_d = t$, accounts for linear growth in nominal money and prices as well as real income. If $\{g_1 \neq 0, g_2 \neq 0, g_3 \neq 0\}$, then $\{E\Delta p_t \neq 0, E\Delta m_t \neq 0, E\Delta y_t \neq 0\}$, i.e. real and nominal growth is nonzero consistent with stylized facts in most industrialized countries. If $g_3 = 0$ and $d_{31} = 0$ in (4.6), then Σu_{2i} is likely to describe the long-run real growth in the economy, i.e. a "structural" unit root process as discussed in the many papers on the stochastic versus deterministic real growth models. See for instance, King, Plosser, Stock and Watson (1991). If $g_3 \neq 0$, then the linear time trend is likely to capture the long-run trend and Σu_{2i} will describe the medium-run deviations from this trend, i.e. the long business cycles. The trend-adjusted real income variable in the middle panel of Figure 4.1. illustrates such long business cycles. For a further discussion, see Rubin (1998). The first case explicitly assumes that the average real growth rate is zero whereas the latter case does not. Whether one includes a linear trend or not in (4.6) influences, therefore, the possibility of interpreting the second stochastic trend, Σu_{2i}, as a long-run structural trend or not.

4.1.1. Long-run price homogeneity

I will take a closer look at the trend components of m_t and p_t in (4.6):

$$\begin{aligned} m_t &= c_{11}\Sigma\Sigma u_{1i} + d_{11}\Sigma u_{1i} + d_{12}\Sigma u_{2i} + g_1 t + stat.\, comp. \\ p_t &= c_{21}\Sigma\Sigma u_{1i} + d_{21}\Sigma u_{1i} + d_{22}\Sigma u_{2i} + g_2 t + stat.\, comp. \end{aligned}$$

If $(c_{11}, c_{21}) \neq 0$, then $\{m_t, p_t\} \sim I(2)$. If, in addition, $c_{11} = c_{21}$ then

$$m_t - p_t = (d_{11} - d_{21})\Sigma u_{1i} + (d_{12} - d_{22})\Sigma u_{2i} + (g_1 - g_2)t + stat.comp.$$

is at most $I(1)$. If $\{(d_{11}-d_{21}) \neq 0, (d_{12}-d_{22}) \neq 0\}$, then m_t and p_t are cointegrating from $I(2)$ to $I(1)$, i.e. they are $CI(2,1)$. If, in addition, $(g_1 - g_2) \neq 0$, then real money stock grows around a linear trend.

The case $(m_t - p_t) \sim I(1)$ implies long-run price homogeneity and is a testable hypothesis. Money stock and prices are moving together in the long-run, but not necessarily in the medium-run (over the business cycle). Long-run and medium-run price homogeneity requires $\{c_{11} = c_{21}$, and $d_{11} = d_{21}\}$, i.e. the AD shocks u_{1t} affect nominal money and prices in the same way both in the long run and in the medium run. Because the real stochastic trend Σu_{2i} is likely to enter m_t but not necessarily p_t, testing long- and medium-run price homogeneity jointly is not equivalent to testing $(m_t - p_t) \sim I(0)$. Therefore, the joint hypothesis is not as straightforward to test as long-run price homogeneity alone.

Note that $(m_t - p_t) \sim I(1)$ implies $(\Delta m_t - \Delta p_t) \sim I(0)$, i.e. long-run price homogeneity implies cointegration between price inflation and money growth. If this is the case, then the stochastic trend in inflation can equally well be measured by the growth in money stock.

4.1.2. Assuming long-run price homogeneity

In the following I will assume that $c_{11} = c_{21}$, and discuss various cases where medium-run price homogeneity is either present or absent.

$$\begin{bmatrix} m_t - p_t \\ \Delta p_t \\ y_t \\ R_{mt} \\ R_{bt} \end{bmatrix} = \begin{bmatrix} d_{11} - d_{21} & d_{12} - d_{22} \\ c_{21} & 0 \\ d_{31} & d_{32} \\ d_{41} & d_{42} \\ d_{51} & d_{52} \end{bmatrix} \begin{bmatrix} \Sigma u_{1i} \\ \Sigma u_{2i} \end{bmatrix} + \begin{bmatrix} g_1 - g_2 \\ 0 \\ g_3 \\ 0 \\ 0 \end{bmatrix} [t] + ... \tag{4.7}$$

In (4.7) all variables are at most $I(1)$. The inflation rate (measured by Δp_t or Δm_t) is only affected by the once cumulated AD trend, Σu_{1i}, but all the other variables can in principle be affected by both stochastic trends, Σu_{1i} and Σu_{2i}.

The case trend-adjusted $(m_t - p_t) \sim I(0)$ requires that both $d_{11} = d_{21}$ and $d_{12} = d_{22}$, which is not very likely from an economic point of view. A priori, one would expect the real stochastic trend Σu_{2i} to influence money stock (by increasing the transactions, precautionary and speculative demands for money) but not the price level, i.e. that $d_{12} \neq 0$ and $d_{22} = 0$.

The case $(m_t - p_t - y_t) \sim I(0)$, i.e. money velocity of circulation is a stationary variable, requires that $d_{11} - d_{21} - d_{31} = 0$, $d_{12} - d_{22} - d_{32} = 0$ and $g_1 - g_2 - g_3 = 0$. If $d_{11} = d_{21}$ (i.e. medium run price homogeneity), $d_{22} = 0$ (real stochastic growth does not affect prices), $d_{31} = 0$ (medium-run price growth does not affect real income), and $d_{12} = d_{32}$, then $m_t - p_t - y_t \sim I(0)$. In this case real money stock and real aggregate income share one common trend, the real stochastic trend Σu_{2i}. The stationarity of money velocity, implying common movements in money, prices, and income, is then consistent with the conventional monetarist assumption as stated by Friedman (1970) that "inflation always and everywhere is a monetary problem". This case, $(m_t - p_t - y_t) \sim I(0)$, has generally found little empirical support (Juselius, 1996, 1998b, Juselius and Gennari, 1998, Juselius and Toro, 1999). As an illustration see the graph of money velocity in the upper panel of Figure 4.2. I will now turn to the more realistic assumption of money velocity being $I(1)$.

The case $(m_t - p_t - y_t) \sim I(1)$, implies that $\{(d_{11} - d_{21} - d_{31}) \neq 0, (d_{12} - d_{22} - d_{32}) \neq 0\}$. It suggests that the two common stochastic trends affect the level of real money stock and real income differently. A few examples illustrate this:

Example 1: Inflation is cointegrating with velocity, i.e.:

$$m_t - p_t - y_t + b_1 \Delta p_t \sim I(0), \tag{4.8}$$

or alternatively

$$(m_t - p_t - y_t) + b_2 \Delta m_t \sim I(0).$$

Under the previous assumptions that $d_{31}, d_{22} = 0$, and $d_{12} = d_{32}$ the $I(0)$ assumption of (4.8) implies that $d_{11} - d_{21} = b_1 c_{21}$. If $b_1 > 0$, then (4.8) can be interpreted as a money demand relation, where the opportunity cost of holding money relative to real stock is a determinant of money velocity. On the other hand if $b_1 < 0$ (or $b_2 > 0$), then inflation adjusts to excess money, though if $\mid b_1 \mid < 1$, with some time lag. In this case it is not possible to interpret (4.8) as a money demand relation.

Example 2: The interest rate spread and velocity are cointegrating, i.e.:

$$(m_t - p_t - y_t) - b_3(R_m - R_b)_t \sim I(0). \tag{4.9}$$

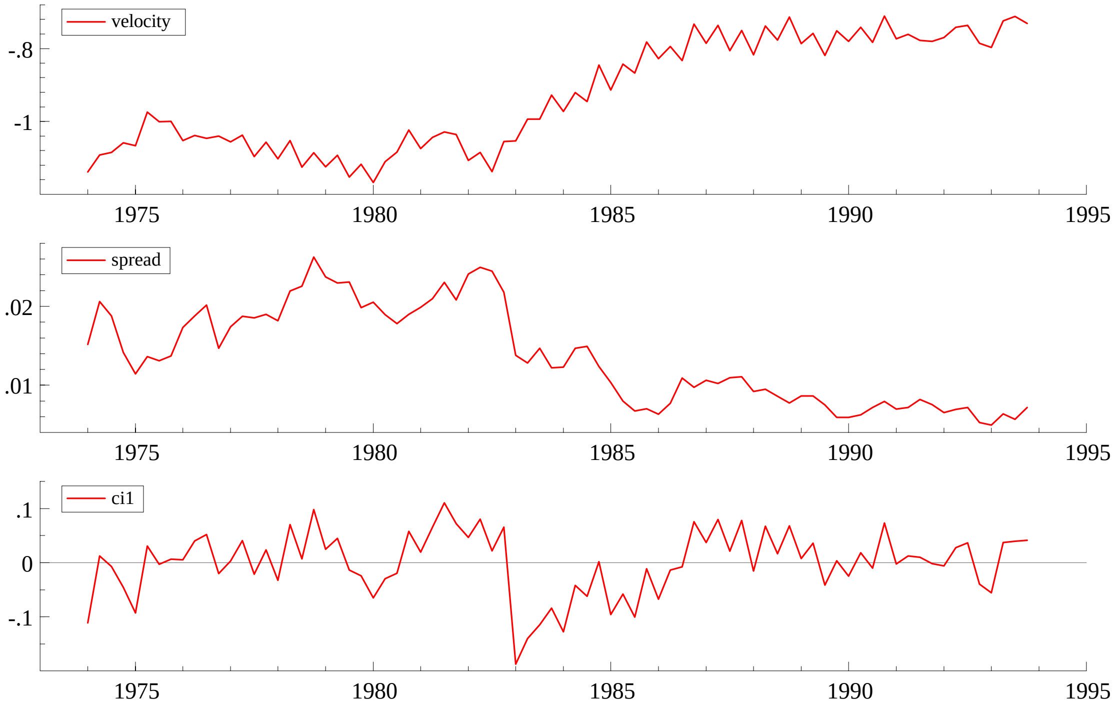

Figure 4.2: Money velocity (upper panel), the interest rate spread (middel panel), and money demand (lower panel) for Danish data.

Because $(R_m - R_b)_t \sim I(1)$, either $(d_{41} - d_{51}) \neq 0$, or $(d_{42} - d_{52}) \neq 0$, or both. In either case the stochastic trend in the spread has to cointegrate with the stochastic trend in velocity. If $b_3 > 0$, then (4.9) can be interpreted as a money demand relation in which the opportunity cost of holding money relative to bonds is a determinant of agent's desired money holdings. On the other hand, if $b_3 < 0$ then the money demand interpretation is no longer possible, and (4.9) could instead be a central bank policy rule. Figure 4.2., the middle panel, shows the interest spread between the Danish 10 year bond rate and the deposit rate, and the lower panel the linear combination (4.9) with $b_3 = 14.1$. It is notable how well the nonstationary behavior of money velocity and the spread cancels in the linear money demand relation.

From the perspective of monetary policy a nonstationary spread suggests that the short-term central bank interest rate can be used as an instrument to influence money demand. A stationary spread on the other hand signals fast adjustment between the two interest rates, such that changing the short interest rate only changes the spread in the very short run and, hence, leaves money demand essentially unchanged.

In a model explaining monetary transmission mechanisms, the determination of real interest rates is likely to play an important role. The Fisher parity predicts that real interest rates are constant, i.e.

$$R_t = \mathcal{E}_t \Delta p_{t+m} \tag{4.10}$$

where $\mathcal{E}_t \Delta p_{t+m}$ is the expected value at time t of inflation at the period of maturity $t+m$.

If $(\Delta p_{t+m} - \mathcal{E}_t \Delta p_{t+m}) \sim I(0)$, then the predictions do not deviate from the actual realization with more than a stationary error. If, in addition, $(\Delta p_t - \Delta p_{t+m}) \sim I(0)$, then $R_t - \Delta p_t$ is stationary. From (4.7) it appears that if $(R_m - \Delta p) \sim I(0)$ and $(R_b - \Delta p) \sim I(0)$, then $d_{42} = d_{52} = 0$. Also if $d_{42} = d_{52} = 0$, then R_m and R_b must be cointegrating, $(R_m - b_4 R_b)_t \sim I(0)$ with $b_4 = 1$ for $d_{41} = d_{51}$. In this sense stationary real interest rates are both econometrically and economically consistent with the spread and the velocity being stationary. It corresponds to the situation where real income and real money stock share the common AS trend, Σu_{2i}, and inflation and the two nominal interest rates share the AD trend, Σu_{1i}. This case can be formulated as a restricted version of (4.7):

$$\begin{bmatrix} m_t - p_t \\ \Delta p_t \\ y_t \\ R_{mt} \\ R_{bt} \end{bmatrix} = \begin{bmatrix} 0 & d_{12} \\ c_{21} & 0 \\ 0 & d_{12} \\ c_{21} & 0 \\ c_{21} & 0 \end{bmatrix} \begin{bmatrix} \Sigma u_{1i} \\ \Sigma u_{2i} \end{bmatrix} + ... \tag{4.11}$$

Though appealing from a theory point of view, (4.11) has not found much empirical support. Instead, real interest rates, interest rate spreads, and money velocity have frequently been found to be nonstationary. This suggests the presence of real and nominal interaction effects, at least over the long business cycle.

By modifying some of the assumptions underlying the Fisher parity, the nonstationarity of real interest rates can be justified. For example, if agents systematically mispredict future inflation, i.e. $(\Delta p_{t+m} - \mathcal{E}_t \Delta p_{t+m}) \sim I(1)$, or if the inflation differential is nonstationary, i.e. $(\Delta p_t - \Delta p_{t+m}) \sim I(1)$, then $R_t - \Delta p_t \sim I(1)$ is consistent with the Fisher parity. In this case one would also expect

$E_t(\Delta p_{t+b} - \Delta p_{t+m}) \sim I(1)$, and $(R_m - R_b)_t \sim I(1)$ would be consistent with the predictions from the expectation's hypothesis (or the Fisher parity).

In Section 7 I will briefly discuss the case where the spread, the real interest rates and the velocity are nonstationary, but otherwise restrict the discussion to the more straightforward case (4.11).

4.2. Treating prices as I(1)

In this case $\rho < 1$ in (4.4) implying that inflation is stationary albeit strongly autocorrelated. The representation of the vector process becomes:

$$\begin{bmatrix} m_t \\ p_t \\ y_t \\ R_{mt} \\ R_{bt} \end{bmatrix} = \begin{bmatrix} c_{11} & d_{12} \\ c_{21} & d_{22} \\ 0 & d_{32} \\ 0 & d_{42} \\ 0 & d_{52} \end{bmatrix} \begin{bmatrix} \Sigma u_{1i} \\ \Sigma u_{2i} \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ 0 \\ 0 \end{bmatrix} [t] + \text{stat.comp.} \tag{4.12}$$

Money and prices are represented by:

$$m_t = c_{11}\Sigma u_{1i} + d_{12}\Sigma u_{2i} + g_1 t + stat.comp.$$

$$p_t = c_{21}\Sigma u_{1i} + d_{22}\Sigma u_{2i} + g_2 t + stat.comp.$$

If $c_{11} = c_{21}$ there is long-run price homogeneity, but $(m_t - p_t) \sim I(1)$ unless $(d_{12} - d_{22}) = 0$. If $d_{12} \neq 0$ and $d_{22} = 0$, then

$$m_t - p_t = d_{12}\Sigma u_{2i} + (g_1 - g_2)t + stat.comp.$$

If $d_{12} = d_{32}$ then $(m_t - p_t - y_t) \sim I(0)$. From $\{m_t,\, p_t\} \sim I(1)$ it follows that $\{\Delta p, \Delta m\} \sim I(0)$, and real interest rates cannot be stationary unless $d_{42} = d_{52} = 0$. Hence, a consequence of treating prices as $I(1)$ is that nominal interest rates should be treated as $I(0)$, unless one is prepared *a priori* to exclude the possibility of stationary real interest rates.

As discussed above, the inflation rate and the interest rates have to cross their mean path fairly frequently to obtain statistically significant mean-reversion. The restricted version of (4.12) given below is economically as well as econometrically consistent, but is usually only relevant in the analysis of long historical data sets.

$$
\begin{bmatrix} m_t \\ p_t \\ y_t \\ R_{mt} \\ R_{bt} \end{bmatrix} = \begin{bmatrix} c_{11} & d_{12} \\ c_{21} & 0 \\ 0 & d_{12} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Sigma u_{1i} \\ \Sigma u_{2i} \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ 0 \\ 0 \end{bmatrix} [t] + \text{stat.comp.} \tag{4.13}
$$

5. Estimation and hypotheses formulation

The economic relations discussed in Section 2 and the stochastic formulation of the macroeconomic variables discussed in Section 4 can be analyzed in a formal statistical framework using the cointegrated VAR model given by:

$$
\begin{gathered} \Delta^2 \mathbf{x}_t = \Pi_1 \Delta^2 \mathbf{x}_{t-1} + \Gamma \Delta \mathbf{x}_{t-1} + \Pi \mathbf{x}_{t-1} + \mu_0 + \mu_1 t + \varepsilon_t, \\ \varepsilon_t \sim N_p(0, \Sigma\ ),\ t = 1, ..., T \end{gathered} \tag{5.1}
$$

where $\mathbf{x}_t$ is a $p \times 1$ vector of variables in the system, the lag length has been set to three, and the parameters $\{\Gamma_1, \Pi_1, \Pi, \mu_0, \mu_1, \Sigma\}$ are unrestricted.

The VAR model is essentially based on the assumption of multivariate normal disturbances, i.e. residuals should behave approximately as a multivariate normal process. In the VAR formulation this amounts to:

$$
\Delta \mathbf{x}_t - E_{t-1}\{\Delta \mathbf{x}_t \mid \mathbf{X}_{t-1}\} = \boldsymbol{\varepsilon}_t \tag{5.2}
$$

where $\mathbf{X}_{t-1} = [\mathbf{x}_{t-1}, \mathbf{x}_{t-2}, \mathbf{x}_{t-3}]$, and $E_{t-1}\{\Delta \mathbf{x}_t \mid \mathbf{X}_{t-1}\}$ is the conditional expectation of $\Delta \mathbf{x}_t$ given the information at time $t-1$. When $\mathbf{x}_t$ contains the most important variables needed to explain the variation in the vector process, the multivariate normality assumption has often shown to work reasonably well. Nevertheless, major reforms, interventions, and extraordinary events, such as joining or leaving the ERM, abolishing restrictions on capital movements, changing wage indexation schemes, extraordinary increases in the oil price, are likely to violate the normality assumption. In such cases the information set has to be enlarged with additional variables accounting for these effects, usually in the form of dummy variables, $\mathbf{D}_t$, i.e.:

$$
\Delta \mathbf{x}_t - E_{t-1}\{\Delta \mathbf{x}_t \mid \mathbf{D}_t, \mathbf{X}_{t-1}\} = \boldsymbol{\varepsilon}_t \tag{5.3}
$$

Note that $\mathbf{D}_t$ in (5.3) enters the information set only at the time the effect was unanticipated. After the intervention has occurred and the effect is known, the adjustment mechanisms of the model should bring the system back to the new steady-state position. For an illustration see the graph in Figure 2.1, lower panel. In some cases, when interventions or reforms change the data generating mechanism, i.e. the parameters of the VAR model, such an assumption is too simple. But in order to simplify the subsequent discussion I will ignore altogether this important issue and assume no structural breaks and no need for intervention dummies in model (5.1). The interested reader is referred to the large econometric literature dealing with regime shifts and structural breaks.

The unrestricted VAR model (5.1) essentially summarizes the first two moments of the data and is, therefore, a statistical description rather than an economic model. By imposing statistically valid and economically interpretable parameter restrictions on the VAR a more interesting model can be found. See for instance Hendry and Mizon (1993) and Hendry (1995) for a discussion of the "general to specific" VAR approach. In Section 5.1 I will discuss restrictions on the VAR parameters that define the I(2) model and in Section 5.2 that define the I(1) model.

5.1. The I(2) model

The empirical justification for nominal money and prices containing a second order stochastic trend and a first order deterministic time trend was discussed in Sections 3 and 4.1. The hypothesis that $\mathbf{x}_t$ is $I(2)$ can be formulated statistically as two reduced rank hypotheses on Π and Γ (Johansen, 1995a), whereas the hypothesis that $\mathbf{x}_t$ contains deterministic linear trends, but no higher order trends, can be formulated as restrictions on μ_0 and μ_1 (Rahbek, Kongsted, and Jorgensen, 1998).

The prior assumption that there are two stochastic trends derived from permanent shocks to the AD and the AS curve is formulated as the hypothesis that $p - r = 2$, where r is the rank of Π. The hypothesis that the nominal trend is $I(2)$ is formulated as an additional reduced rank hypothesis on Γ. Formally this is stated as: $\Pi = \alpha\beta'$ and $\alpha'_{\perp}\Gamma\beta_{\perp} = \zeta\eta'$, where α, β are $p \times r$ matrices, $\alpha_{\perp}, \beta_{\perp}$are $p \times (p - r)$ matrices orthogonal to α and β, and ζ, η are $p - r \times s_1$ matrices and s_1 is the number of $I(1)$ trends (Johansen, 1991). Hence, the reduced rank of Π is related to the total number of stochastic trends in the data, $p - r$, whereas the reduced rank of Γ, s_1,is related to the number of second order nonstationary trends in the data, s_2. Hence, $p - r = s_1 + s_2$.

By assuming $\Pi = \alpha\beta'$ and $\alpha'_{\perp}\Gamma\beta_{\perp} = \zeta\eta$ and zero restrictions on quadratic or higher order trends we can solve the VAR with respect to ε_t and deterministic components to derive the model in moving average form:

$$\mathbf{x}_t = \mathcal{B}_2\mathcal{A}_2\prime \sum_{s=1}^{t}\sum_{i=1}^{s}\varepsilon_i + \mathcal{B}_1\mathcal{A}_1\prime \sum_{s=1}^{t}\varepsilon_s + gt + Y_t + A + Bt,\ t = 1, ..., T \tag{5.4}$$

where $\mathcal{B}_2, \mathcal{A}_2$ are $p \times s_2$ matrices, $\mathcal{B}_1, \mathcal{A}_1$ are $p \times (p-r)$ matrices, g are the linear trends in the data, Y_t defines the stationary part of the process, and A and B are a function of the initial values $x_0, x_{-1}, ..., x_{-k+1}$. See Johansen (1995a,1995c) and Paruolo (1996) for further details. In this form it is straightforward to interpret $\mathcal{A}_2\prime\varepsilon_t = u_{1t}$ as the shock leading to the $I(2)$ trend $\Sigma\Sigma\mathcal{A}_2\prime\varepsilon_i = \Sigma\Sigma u_{1i}$, and $\mathcal{A}_1\prime\varepsilon_t = u_t$, where $u_t' = [u_{1t}, u_{2t}]$, as the two $I(1)$ trends Σu_{1i} and Σu_{2i}. The derivation of $\{\mathcal{B}_2, \mathcal{A}_2, \mathcal{B}_1, \mathcal{A}_1, g\}$ as a function of the parameters $(\Pi_1, \Gamma, \alpha, \beta, \mu_0, \mu_1, \Sigma)$ can be found in Johansen (1995a).

Hence, the appropriately restricted VAR model can reproduce the decomposition into stochastic trends, cycles, irregular components (4.7) discussed in Section 4. Because the VAR is a statistically well-specified model, the basic assumptions underlying the empirical problem such as (i) the number and (ii) order of the stochastic trends, (iii) the presence of linear trends or (iv) quadratic trends in the data, can be checked against the data based on strict hypothesis testing. From the outset of the empirical investigation it is, therefore, possible to infer from the data if the basic economic arguments underlying the economic model are statistically acceptable. If they are not, then the empirical results are likely to suggest in which directions the theoretical framework should be modified. In this sense a stringent VAR analysis can be a useful complement to conventional economic analysis, by solving the problem of having to rely on basic assumptions never tested.

5.2. The I(1) model.

Section 4 showed that long-run price homogeneity implies cointegration between nominal money stock m and prices p. In this case a reformulation of the data vector based on real money and inflation (instead of nominal money and prices) transforms the model to $I(1)$. This can be formulated as the hypothesis that $\mathbf{x}_t = [m-p, y, \Delta p, R_m, R_b]$ is $I(1)$.

The $I(1)$ model is statistically defined by the reduced rank of $\Pi = \alpha\beta'$ and the full rank of $\alpha_\perp'\Gamma\beta_\perp$. Thus, the $I(1)$ model is based on one reduced rank condition, whereas the $I(2)$ model was based on two. Rewriting (5.1) in first differences and levels leads to the well-known cointegrated VAR model in error correction form:

$$\begin{aligned} \Delta \mathbf{x}_t &= \Gamma_1 \Delta \mathbf{x}_{t-1} + \alpha\beta' \mathbf{x}_{t-1} + \mu_0 + \varepsilon_t, \\ \varepsilon_t &\sim N_p(0, \Sigma\ ),\ t = 1, ..., T \end{aligned} \tag{5.5}$$

where $\Gamma_1 = I - \Gamma$ and $\Pi_1 = 0$, i.e. we have assumed only two lags in the model. Solving (5.5) for ε_t and the deterministic terms gives the moving average representation of the $I(1)$ model:

$$\mathbf{x}_t = \mathcal{B}_1 \mathcal{A}_1' \sum_1^t \varepsilon_i + \mathcal{B}_1 \mathcal{A}_1' \mu_0 t + C^*(L)(\varepsilon_t + \mu) + B \tag{5.6}$$

where $\mathcal{B}_1 \mathcal{A}_1' = \beta_\perp (\alpha_\perp' \Gamma \beta_\perp)^{-1} \alpha_\perp' = C$, $\mathcal{B}_1 \mathcal{A}_1' \mu_0 = g$, and $C^*(L)\varepsilon_t$ describes the stationary part of the vector process as an infinite polynomial in the lag operator L, and B is a function of the initial values (Johansen, 1991). The lag polynomial $C^*(L)\varepsilon_t$ can describe stationary cycles of short duration, whereas (as discussed in Section 4) $\mathcal{A}_1' \sum_1^t \varepsilon_i$ can describe both long-run and medium-run stochastic trends, where the latter would corrspond to long business cycles. It appears, therefore, that (5.6) reproduces the composition into trend, cycle, irregular component as given by (4.7) in Section 4. By choosing $\mathcal{A}_1 = \alpha_\perp$ an estimate of the common stochastic trends are obtained:

$$u_t = \alpha_\perp' \sum_1^t \varepsilon_i \tag{5.7}$$

Although $\Pi = \alpha\beta'$ and $C = \mathcal{B}_1 \mathcal{A}_1'$ are uniquely determined, the decomposition into α, β and $\mathcal{B}_1, \mathcal{A}_1$ are not, as demonstrated by:

$$\begin{aligned} \alpha\beta' = \alpha H H^{-1\prime} \beta' = \widetilde{\alpha}\widetilde{\beta}' = \Pi \\ \mathcal{B}_1 \mathcal{A}_1' = \mathcal{B}_1 Q Q^{-1\prime} \mathcal{A}_1' = \widetilde{\mathcal{B}}_1 \widetilde{\mathcal{A}}_1' = C \end{aligned}$$

where H is a $r \times r$ and Q a $(p-r) \times (p-r)$ nonsingular matrix. In addition, as will be discussed in Section 9 and 10 the short-run adjustment parameters and the residuals ε_t are invariant to linear transformation of (5.5). Therefore, after the basic structure of the VAR model has been determined, the actual economic modelling consists of how to impose identifying and over-identifying restrictions on the model parameters. This will be further discussed in Sections 7, 8, and 9.

6. On the choice of rank

The cointegration rank divides the data into r adjusting and $p-r$ non-adjusting components. The former are often given an interpretation as equilibrium errors (deviations from steady-state) and the latter as common driving trends in the system. Hence, the choice of r is crucial for all subsequent econometric analysis and for inference on economic hypotheses. The LR trace test (Johansen, 1995c) for the determination of the number of unit roots (stochastic trends) in the vector process is based on asymptotic distributions that depends on the deterministic terms in the VAR model. Asymptotic distributions have been tabulated for the most frequently used alternatives, such as a constant and a trend in the model, and restricted versions of them. When model (5.1) contains institutional dummies that cumulate to broken trends in the DGP, the available asymptotic tables are no longer valid (Johansen and Nielsen 1993) and each case should be tabulated separately.

As many simulation studies have demonstrated, the asymptotic distributions are often rather poor approximations in small samples, with consequent size and power distortions (Johansen, 1998, 1999, Jørgensen, 1999). Because empirical macroeconomic models are typically based on sample sizes of 50-100 observations caution is definitely needed.

The trace test procedure consists of testing the hypothesis "at least one unit root" and if accepted continues until the first rejection occurs at "at least $p-r+1$ unit roots". This means that the procedure is essentially based on the principle of "no prior economic knowledge " regarding the rank r. This is in many cases difficult to justify. As demonstrated in Section 4 and 5 in the money market example, the case ($r=3$, $p-r=2$) can be considered plausible for a reasonably deregulated economy. For a more regulated economy we might have slower market adjustment and, hence, the case ($r=2$, $p-r=3$) might be preferable *a priori*.

An alternative procedure is, therefore, to test a given prior economic hypothesis, say $p-r=2$, using the trace test and, if accepted, continue with this assumption unless the data strongly suggests the presence of additional unit roots. This can be investigated for example by testing the significance of the adjustment coefficients α_{ir}, $i=1,...,p$ to the $r'th$ cointegrating vector. If all α_{ri} coefficients are insignificant, then including the $r'th$ cointegrating relation in the model would not improve the explanatory power of the model but, would in fact invalidate subsequent inference. Additionally, if the choice of r incorrectly includes a non-stationary relation among the cointegrating relations, then one of the roots of the characteristic polynomial of the model is a unit root or a near unit root. If either of these cases occur, then the cointegration rank should be reduced. Note, however, that additional unit roots in the characteristic polynomial can be the consequence of $I(2)$ components in the data vector. In this case reducing the rank will not solve the problem.

Note, however, that the hypothetical cointegration rank is not in general equivalent to the number of theoretical steady-state relations derived from a partial economic model. For instance, in the money market example of Section 2 there was one equilibrium relation (2.1). As demonstrated in Section 4 and 5 (with two instead of just one interest rate) this is consistent with $r = 3$ cointegrating relations and not $r = 1$, which has been incorrectly assumed in many empirical applications. Hence, cointegration between variables is a statistical property of the data that only exceptionally can be given a direct interpretation as an economic steady-state relation.

7. Restrictions on the long-run parameters

Given the number of cointegrating relations, r, the Johansen procedure gives the maximum likelihood estimates of the unrestricted cointegrating relations $\beta'\mathbf{x}_t$. Although the unrestricted β is uniquely determined based on the chosen normalization of the reduced rank problem, the latter is not necessarily meaningful from an economic point of view. Therefore, an important part of the empirical analysis is to impose (over-) identifying restrictions on β to achieve economic interpretability. As an example of just identifying restrictions, consider the following design matrix $Q = [\beta_1]$ where β_1 is a $(r \times r)$ nonsingular matrix defined by $\beta' = [\beta_1, \beta_2]$. In this case $\alpha\beta' = \alpha(\beta_1\beta_1^{-1\prime}\beta') = \alpha[I, \widetilde{\beta}]$ where I is the $(r \times r)$ unit matrix and $\widetilde{\beta} = \beta_1^{-1\prime}\beta_2$ is a $(r \times p - r)$ full rank matrix. These just-identifying restrictions have transformed β to the long-run "reduced form". Because just-identifying restrictions do not change the likelihood function, no tests are involved. In general just identification can be achieved by imposing one normalization and $(r-1)$ restrictions on each β_i. Additional restrictions are overidentifying and, therefore, testable. See Johansen (1995b) and Johansen and Juselius (1992 and 1994) for a more detailed treatment.

Consistent with the discussion in section 4, I will assume three cointegrating relations in the money market example and, hence, two autonomous common trends, Σu_{1i} and Σu_{2i}. Assuming nominal and real separation of the long-run structure as in (4.11) we would expect the following hypothetical cointegrating relations:

- $(m-p-y) \sim I(0)$,
- $(R_b - R_m) \sim I(0)$,
- $(R_b - \Delta p) \sim I(0)$.

Each imposes two overidentifying restrictions on the cointegrating space, i.e. a total of six testable restrictions. As illustrated below they completely determine the cointegration space:

$$\beta'\mathbf{x}_t = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} (m-p)_t \\ y_t \\ \Delta p_t \\ R_{m_t} \\ R_{b_t} \end{bmatrix}.$$

If, however, there are permanent (at least over the business cycle horizon) interaction effects between the nominal and the real side of the economy one would expect the above relations to be $I(1)$:

- $(m-p-y) \sim I(1)$
- $(R_b - R_m) \sim I(1)$
- $(R_b - \Delta p) \sim I(1)$.

Section 4 demonstrated that when real money stock and real income are not cointegrating, then one of them, or both, must be affected by the stochastic nominal trend. Similarly, when the two interest rates are not cointegrating, then at least one of the interest rates must be affected by the two stochastic trends. This might suggest market rigidities or imperfections, as the following example illustrates. Assume that $R_b - \Delta p \sim I(0)$, but $R_m - \Delta p \sim I(1)$. Moreover, assume that the bond rate is affected only by the stochastic inflation trend, Σu_{1i}, but that the short-term rate is affected by the inflation trend, Σu_{1i}, as well as the long business cycle trend, Σu_{2i}. In this case $(R_b - R_m) \sim I(1)$ could be consistent with an economy where the central bank determines the short rate, based on a policy rule that includes excess inflation as well as excess aggregate demand, i.e. $R_s = f(\Delta p - \pi^*, y - \beta t)$, where π^* is a fixed target rate, but the bond rate is determined by $R_b = f(\Delta p)$.

The following cointegration relations might suggest the presence of long lasting real and nominal interaction effects in the money market:

- $\{(m-p-y)+\beta_1(R_b-R_m)\} \sim I(0)$
- $\{R_m-R_b-\beta_2\Delta p\} \sim I(0)$
- $(y-\beta_3\Delta p-\beta_4 t) \sim I(0)$

With some minor modifications, the stationarity of the above relations has found surprisingly strong empirical support in many empirical applications. See for example Juselius (1998a, 1998b), Gennari and Juselius (1998), and Juselius and Toro (1998), all of them analyzing European money markets based on two decades of quarterly data for Germany, Denmark, Italy and Spain. The stationarity of the first relation was illustrated in Section 4, Figure 4.2, where velocity and the interest rate spread were shown to contain the same stochastic trend, so that the linear combination, $(m-p-y)_t-14(R_m-R_b)_t$, became a stationary steady-state relation.

The above examples have shown that a money demand relation of the type discussed in Romer (1996) can be found empirically either as a linear combination of two stationary cointegration relations $(m-p-y)_t$ and $(R_m-R_b)_t$, or as a cointegration relation between two nonstationary processes, $(m-p-y)_t-b_2(R_m-R_b)_t$. The implications of either case for the effectiveness of monetary policy may, however, be quite different.

For this reason, cointegration properties can provide useful information about the relationship between, say, a policy instrument variable and a target or a goal variable. These relationships are quite likely to change as the economy becomes deregulated, thereby allowing market forces to become more effective. Therefore, changes in cointegration properties can contain valuable information about the consequences of shifting from one regime to another. See for instance Juselius (1998a).

8. Restrictions on the common trends

As discussed in Section 5, for each α and β describing long-run relations in the data and the adjustment towards them, there is a dual representation in terms of $\alpha_\perp$ and $\beta_\perp$ that describes the common trends and their loadings. The $p\times(p-r)$ matrix $\alpha_\perp$ describing the common stochastic trends, $\alpha'_\perp\Sigma\varepsilon_i$, is overparametrized in the sense that it is possible to impose one normalization and $(p-r-1)$ restrictions on each $\alpha_{\perp i}$, without changing the likelihood function. Additional restrictions are overidentifying and, hence, testable.

There are essentially two identification problems in the common stochastic trends case: the first is how to impose restrictions on the residuals $\hat{\varepsilon}_t$; the second is how to impose restrictions on the coefficients of $\alpha_{\perp}$. The first identification problem arises because the VAR residuals are not invariant to linear transformations. This can be illustrated by pre-multiplying (5.5) with the nonsingular $(p \times p)$ matrix A_0 :

$$A_0 \Delta \mathbf{x}_t = A_0 \Gamma_1 \Delta \mathbf{x}_{t-1} + A_0 \alpha \beta' \mathbf{x}_{t-1} + A_0 \mu + A_0 \varepsilon_t, \tag{8.1}$$

or equivalently:

$$\begin{aligned} A_0 \Delta \mathbf{x}_t &= A_1 \Delta \mathbf{x}_{t-1} + a \beta' \mathbf{x}_{t-1} + \mu_a + v_t, \\ v_t &\sim N_p(0, \Omega) \end{aligned} \tag{8.2}$$

where $A_1 = A_0 \Gamma_1$, $a = A_0 \alpha$, $\mu_a = A_0 \mu$, and $v_t = A_0 \varepsilon_t$.

Because different choices of A_0 lead to different estimates of the residuals, the question of how to define a shock is important. The theoretical concept of a shock, and its decomposition into an anticipated and unanticipated part, has a straightforward correspondence in the VAR model as a change of a variable, $\Delta \mathbf{x}_t$, and its decomposition into its explained part, the conditional expectation $E_{t-1}\{\Delta \mathbf{x}_t \mid \mathbf{X}_{t-1}\}$, and its unexplained part, the residual ε_t. The requirement for ε_t to be a correct measure of an unanticipated shock is that the conditional expectation $E_{t-1}\{\Delta \mathbf{x}_t \mid \mathbf{x}_{t-1}\}$ correctly describes how agents form their expectations. For example, if agents use model based rational expectations from a model that is different from the VAR model, then the conditional expectation would no longer be an adequate description of the anticipated part.

Theories also require shocks to be "structural", usually implying that they are, in some sense, objective, meaningful, or absolute. With the reservation that the word structural has been used to cover a wide variety of meanings, I will here assume that it describes a situation when the effect of a shock is (1) unanticipated (novelty), (2) unique (a shock hitting money stock alone), and (3) invariant (no additional explanation by increasing the information set).

As discussed above the argument for a shock to be novel relies on the credibility of the expectations formation, i.e. whether $\varepsilon_t = \Delta \mathbf{x}_t - E_{t-1}\{\Delta \mathbf{x}_t \mid g(\mathbf{X}_{t-1})\}$ is a correct measure of the unanticipated change in x. The uniqueness can be achieved econometrically by choosing A_0 such that the covariance matrix Σ becomes diagonal and, as will be demonstrated below, by appropriately restricting $\alpha_{\perp}$. For empirical applications see Mellander, Vredin and Warne (1992) and Hansen and Warne (1995). But whether the resulting estimate of ε_t can be interpreted as economically novel and unique, depends crucially on the plausibility of the underlying assumptions. Some of them cannot be checked against the data, for example the diagonality of Σ, and different schools will claim structurality for differently derived estimates.

The requirement that a structural shock must not change when increasing the information set, is probably the most crucial from an empirical point of view. Theoretical models, based on which structural uniqueness is claimed, are always based on many simplifying assumptions inclusive the *ceteris paribus* assumption. In empirical models the *ceteris paribus* assumption can only be met by conditioning on omitted theory information. Since most (all) macroeconomic systems are stochastic and highly interdependent, the inclusion of additional variables in the model is likely to change the estimated shocks. See also the discussion in Levtchenkova, Pagan, and Robertson (1998).

Although derived from sophisticated theoretical models, structural interpretability of estimated shocks seems hard to justify. In my view, a structural shock is a theoretical concept with little empirical content in macro-econometric modelling. This does not imply that empirical analyses of common trends based on the estimated residuals $\hat{\varepsilon}_t$ from (5.5) or $\hat{v}_t$ from (8.1) and restricted versions of them are uninteresting. Far from useless they can provide valuable insight concerning the driving forces within the specific system under analysis.

As demonstrated by (5.7) a straightforward estimate of the unrestricted $p-r$ common stochastic trends is given by $\alpha'_{\perp}\Sigma\varepsilon_i$. For example, the unrestricted common trends in the money market example would be:

$$\begin{aligned} u_{1t} &= f(\varepsilon_m, \varepsilon_y, \varepsilon_{\Delta p}, \varepsilon_{Rb}, \varepsilon_{Rm}) = \alpha'_{\perp 1}\Sigma_{i=1}^{t}\hat{\varepsilon}_i, \\ u_{2t} &= f(\varepsilon_m, \varepsilon_y, \varepsilon_{\Delta p}, \varepsilon_{Rb}, \varepsilon_{Rm}) = \alpha'_{\perp 2}\Sigma_{i=1}^{t}\hat{\varepsilon}_i, \end{aligned}$$

They are overidentified in the sense that one can impose $(p-r-1)$ restrictions and a normalization on each vector without changing the likelihood function. Just identification can be achieved by imposing one zero restriction on each vector, for example, by assuming that real shocks do not influence the nominal stochastic trend, and that money shocks have no influence on the real stochastic trend:

$$\begin{aligned} \Sigma\widetilde{u}_{1i} &= \Sigma f(\varepsilon_m, 0, \varepsilon_{\Delta p}, \varepsilon_{Rb}, \varepsilon_{Rm})_i = \widetilde{\alpha}'_{\perp 1}\Sigma_{i=1}^{t}\hat{\varepsilon}_i, \\ \Sigma\widetilde{u}_{2i} &= \Sigma f(0, \varepsilon_y, \varepsilon_{\Delta p}, \varepsilon_{Rb}, \varepsilon_{Rm})_i = \widetilde{\alpha}'_{\perp 2}\Sigma_{i=1}^{t}\hat{\varepsilon}_i. \end{aligned}$$

The overidentifying assumption that the nominal stochastic trend derives solely from shocks to money stock and the real stochastic trend derives from shocks to real income is expressed as:

$$\begin{aligned}\widetilde{\widetilde{u}}_{1t} &= f(\varepsilon_m, 0, 0, 0, 0)\\ \widetilde{\widetilde{u}}_{2t} &= f(0, \varepsilon_y, 0, 0, 0).\end{aligned}$$

This hypothesis imposes three overidentifying restrictions on each vector of $\alpha_\perp$, i.e. altogether six testable restrictions.

9. Restrictions on the short-run parameters

Formulation (8.2) showed that the choice of transformation matrix A_0 does not affect the specification of the long-run parameters β. Hence, one can impose identifying restrictions on the long-run and short-run parameters in two steps. I will, therefore, assume that the β vectors have been fully identified in the first step of the identification scheme. The discussion of how to impose identifying restrictions on the short-run parameters of (8.2) will be based on a given identified $\beta = \widetilde{\beta}$, i.e. for:

$$\begin{aligned}&A_0\Delta\mathbf{x}_t = A_1\Delta\mathbf{x}_{t-1} + a\widetilde{\beta}'\mathbf{x}_{t-1} + \mu_a + v_t\\ &v_t \sim N_p(0, \Omega\ ).\end{aligned} \tag{9.1}$$

The unrestricted set of parameters $(A_0, A_1, a, \mu_a, \Omega)$ is not uniquely determined in the sense that one can impose $(p-1)$ restrictions and one normalization on each equation, without changing the likelihood function. Additional restrictions are overidentifying and, hence, testable (see Johansen and Juselius, 1994). User friendly computer programs are readily available for empirical application of the tests, for example the powerful test procedures in PcFiml (Doornik and Hendry, 1998).

The VAR model is usually heavily overparametrized. This is particularly so for the short-run parameters. By imposing zero restrictions, other linear restrictions, and nonlinear (for example symmetry) restrictions, the number of model parameters can be substantially reduced. The identification problem is essentially about how to satisfy (i) mathematical uniqueness, (ii) statistical significance, and (iii) economic interpretability. In Johansen and Juselius (1994) these three stages are called generic, empirical, and economic identification.

Mathematical uniqueness is related to such restrictions on (9.1) that give a unique mapping between (5.5) and (9.1). Statistical significance is important for two reasons; (i) the coefficients of our empirical model should preferably describe relevant aspects of the investigated economic system, (ii) leaving insignificant coefficients in the model is likely to introduce near singularity in the model. This is particularly so if the insignificant coefficients violate mathematical uniqueness when restricted to zero. The economic interpretability is definitely the most difficult and demanding requirement of the identification process and where cooperation between empirical and theoretical economists is likely to be most rewarding.

It appears from (9.1) that the VAR model describes the anticipated part of a change from $t-1$ to t in money, income, inflation and interest rates by:

- current (anticipated) changes in the system variables,
- short-run adjustment to the lagged changes of the system variables (temporary dynamic effects),
- short-run adjustment to long-run steady-states (the cointegrating relations).

The representation (9.1) allows for the possibility that agents react differently on (i) disequilibrium between levels of variables, (ii) lagged changes in the determinants, and (iii) nominal acceleration rates. This very rich empirical structure allows us to ask relevant economic questions within a framework that mimics the actual behavior in economic systems, such as dynamic adjustment towards long-run and medium-run steady-states, strong interaction effects, etc.

I will end the discussion by first demonstrating how the original question of a money-demand relation raised by Romer (1996) can be addressed within the VAR model. I will then illustrate some extensions of the analysis related to the effectiveness of monetary policy. In doing so I will simplify (9.1) by assuming that A_0 is lower triangular, Ω is diagonal, and $A_1 = 0$. Generalization to more complicated specifications should be straightforward. A detailed specification of the model is given below:

$$\begin{bmatrix} 1 & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & 1 & a_{23} & a_{24} & a_{25} \\ 0 & 0 & 1 & a_{34} & a_{35} \\ 0 & 0 & 0 & 1 & a_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta {m^r}_t \\ \Delta^2 p_t \\ \Delta y_t \\ \Delta R_{mt} \\ \Delta R_{bt} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} +$$

$$
+\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} \end{bmatrix} \begin{bmatrix} (m-p-y)_{t-1} \\ (R_b - R_m)_{t-1} \\ (R_b - \Delta p)_{t-1} \end{bmatrix} + \begin{bmatrix} v_{mt} \\ v_{\Delta pt} \\ v_{yt} \\ v_{R_{mt}} \\ v_{R_{bt}} \end{bmatrix} \tag{9.2}
$$

where the cointegration relations are given by the first set of identified relations discussed in Section 7. Within this framework I will address three questions related to the effectiveness of monetary policy. To simplify the discussion I have chosen to focus solely on the money and inflation equations, albeit being aware that a complete answer should be based on an analysis of the full system.

$$
\begin{array}{lll}
\Delta m_t^r & = & a_{12}\Delta^2 p_t + a_{13}\Delta y_t + a_{14}\Delta R_{m_t} + a_{15}\Delta R_{b_t} + \\
 & & \alpha_{11}(m-p-y)_{t-1} + \alpha_{12}(R_b - R_m)_{t-1} + \alpha_{13}(R_b - \Delta p)_{t-1} + \varepsilon_{mt} \\
\Delta y_t & = & .... \\
\Delta^2 p_t & = & a_{33}\Delta y_t + \gamma_{34}\Delta R_{m_t} + \gamma_{35}\Delta R_{bt} + \\
 & & \alpha_{31}(m-p-y)_{t-1} + \alpha_{32}(R_b - R_m)_{t-1} + \alpha_{33}(R_b - \Delta p)_{t-1} + \varepsilon_{pt} \\
\Delta R_{mt} & = & ..... \\
\Delta R_{bt} & = & .....
\end{array}
$$

Question 1. *Is money stock adjusting to money demand or supply?*

If $\alpha_{11} < 0$, $\alpha_{12} < 0$,and $\alpha_{13} = 0$, then empirical evidence is in favor of money holdings adjusting to a long-run money demand relation. The latter can be derived from the money stock equation as follows:

$$
\begin{aligned} (m-p) &= y + \alpha_{12}/\alpha_{11}(R_b - R_m) \\ &= y + \beta_1(R_b - R_m) \end{aligned} \tag{9.3}
$$

Relation (9.3) corresponds to the aggregate demand-for-money relation discussed in a static equilibrium framework by Romer, with the difference that (9.3) is imbedded in a dynamic framework.

If $\alpha_{11} < 0$, $\alpha_{12} = 0$,and $\alpha_{13} = 0$, empirical evidence is in favor of money holdings adjusting to money supply. This could be consistent with a situation where central banks are able to effectively control money stock and the monetary policy control rule is to keep velocity around a constant level.

Question 2. *Is an empirically stable money demand relation a prerequisite for central banks to be able to control inflation rate?* This hypothesis is based on essentially three arguments:

2.1 There exists an empirically stable demand for money relation.
2.2. Central banks can influence the demanded quantity of money.
2.3. Deviations from this relation cause inflation.

First, the empirical requirement for a stable money demand relation is that $\{\alpha_{11} < 0,\ \alpha_{12} < 0,\text{and } \alpha_{13} = 0\}$, and that the estimates are empirically stable. In this case money stock is endogenously determined by agents' demand for money and it is not obvious that money stock can be used as a monetary instrument by the central bank.

Second, given the previous result (i.e. $\alpha_{11} < 0$, $\alpha_{12} < 0$,and $\alpha_{13} = 0$), central banks cannot directly control money stock. Nevertheless, controlling money stock indirectly might still be possible by changing the short-term interest rate. But $\alpha_{12} < 0$ and $(R_b - R_m) \sim I(0)$ implies that a change in the short-term interest rate will transmit through the system in a way that leaves the spread basically unchanged. Therefore, if the data have the property of (4.11) it seems difficult to justify a claim that central banks can effectively control money stock.

Third, if deviations from a stable money demand relation do cause inflation (in the short-run), then the following condition should be satisfied: $\{\alpha_{31} > 0,\ \alpha_{32} > 0,\text{and } \alpha_{33} = 0\}$. Given the assumption that agents can satisfy their desired level of money, it does not seem plausible that agents would deliberately choose to hold excess money (negative or positive). In this case excess money would lead to inflation only if central banks insist on supplying more money than that demanded (which seems to be the opposite of inflation control). Therefore, the frequently used assumption that an empirically stable demand for money relation is needed for central banks to be able to control inflation seems logically flawed.

Question *3. Is money causing prices or prices causing money?*

When discussing this question one has to distinguish between the case when money stock is (1) demand determined or (2) supply determined. If money stock is demand determined then the arguments above suggests that prices are causing money. If money stock is supply determined and central banks can effectively control money stock, for example by keeping money velocity at a stationary level, then $\{\alpha_{11} < 0, \alpha_{12} = 0,\text{and } \alpha_{13} = 0\}$ should be the case. If inflation is increasing in the short run with excess money supply, then we would expect $\{\alpha_{31} > 0, \alpha_{12}, \alpha_{13} = 0\}$. In this case prices are adjusting in the short run to the level of nominal money stock. The question whether prices cause money or money cause prices in the long run has to be answered based on the estimates of the long-run impact of a shock in money stock on prices, as compared to the impact of a shock in prices on money stock.

Therefore, the question whether money causes prices or prices cause money empirically is a difficult and intricate econometric question. This is in striking contrast to the simplistic argument behind the inverted money demand relation in Romer (1996).

10. Conclusions

The motivation for writing this paper was to point out the need for research programs in macroeconomics that mimics that of the present state of research in macro-econometrics, in the sense of developing theoretical models of the macroeconomy that replicates the basic features of macroeconomic data, such as:

- stochastic variables and relations,
- strongly time dependent data,
- cointegration and integration properties,
- short-run adjustment toward long-run dynamic or static steady-states,
- short-run and long-run feedback effects.

I have tried to illustrate the potential usefulness of the cointegrated VAR model to address important macroeconomic questions. My purpose has been to demonstrate that within this framework:

- conventional macroeconomic questions can be efficiently addressed,
- a large number of additional questions can be empirically investigated,
- new macroeconomic hypotheses are likely to emerge for further theoretical development.

I have primarily addressed questions related to the cointegration property of the data and the corresponding dynamic adjustment processes, thereby demonstrating the potential usefulness of the VAR model for inference on macroeconomic transmission mechanisms. But a large number of additional questions related to the analysis of static and dynamic steady-states, speed of adjustment, long-run and short-run feed-back effects, weak, strong, and super exogeneity, driving forces, expectations formation, etc., have only been briefly touched upon. The interested reader is referred to a large number of published papers that address these issues in more detail.

With the above (admittedly quite simplistic) examples I have tried to demonstrate that careful empirical analyses based on the basic features of the cointegrated VAR model can be useful for asking relevant macroeconomic questions. The requirement that all empirical statements have to be consistent with the information given by the full stochastic model, should minimize the risk for "ad hoccery" results. Nevertheless, data do play a dominant role in this approach and the ultimate testing of new hypotheses suggested by the empirical analysis has to be made against new data.

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